Mark Scheme (Results)

Summer 2016

Pearson Edexcel IAL in Core Mathematics 34 (WMA02/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**PEARSON EDEXCEL IAL MATHEMATICS**

## General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Core Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles).*

**Method mark for solving 3 term quadratic:**

### 1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

### 2. Formula

Attempt to use the correct formula (with values for a, b and c).

### 3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

**Method marks for differentiation and integration:**

### 1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

### 2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question Number | Scheme | Notes | Marks |
| --- | --- | --- | --- |
| **1.(a)** | $R = \sqrt{34}$ | Cao (Must be exact but score when first seen and ignore decimal value (5.83…)) | B1 |
| | $\tan\alpha = \pm\frac{5}{3}$, $\tan\alpha = \pm\frac{3}{5} \Rightarrow \alpha = ...$ <br> (Allow $\cos\alpha = \pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}$, $\sin\alpha = \pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}} \Rightarrow \alpha = ...$) <br> Where $\sqrt{34}$ is their $R$ | | M1 |
| | $\alpha = 59.04°$ | awrt 59.04° | A1 |
| | | | **(3)** |
| **(b)** | $\sqrt{34}\cos(\theta - 59.04) = 2 \Rightarrow \cos(\theta - 59.04) = \frac{2}{\sqrt{34}}(0.343)$ <br> Attempts to use part (a)$"\sqrt{34}"\cos(\theta - "59.04") = 2$ and proceeds to $\cos(\theta \pm "59.04") = K$, $\lvert K\rvert \leqslant 1$ <br> May be implied by $\theta - "59.04" = 69.94...^{\circ}$ or $\theta - "59.04"\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)$ <br> The $\theta - "59.04"$ must be seen here or implied later | | M1 |
| | $\theta_1 - 59.04 = 69.94 \Rightarrow \theta_1 = \text{awrt } 129.0°$ | | A1 |
| | $\theta_2 \pm 59.04 = 360 - '69.94' \Rightarrow \theta_2 = ...$ <br> Correct attempt at a second solution in the range. <br> It is **dependent** upon having scored the previous M. <br> Usually for $\theta -$ their $59.04 = 360 -$ their $'69.94' \Rightarrow \theta = ...$ | | **d**M1 |
| | $\theta_2 = 349.1°$ | awrt 349.1° | A1 |
| | For solutions in (b) that are otherwise fully correct, if there are extra answers in range, deduct the final A mark. | | |
| | | | **(4)** |
| **(c)** | $\theta + \text{their } 59.04 = \cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right) \Rightarrow \theta = ...$ <br> Allow $\theta - \text{their } 59.04 = \cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right) \Rightarrow \theta = ...$ if they have $\theta +$ .. in (b) <br> Evidence that use is being made of parts (a) and (b) to obtain a value for $\theta$. This can be implied by the use of their answers to part (b). | | M1 |
| | $\theta = 10.9^{\circ}$ | awrt 10.9 | A1 |
| | | | **(2)** |
| | | | **(9 marks)** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **2** | $\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x}=4x\cos x+4\sin x$ | Applies product rule to $4x\sin x$ to give $\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x}=\pm 4x\cos x+4\sin x$ | M1 |
| | $\frac{\mathrm{d}(\pi y^2)}{\mathrm{d}y}=2\pi y\frac{\mathrm{d}y}{\mathrm{d}x}$ | Applies chain rule to $\pi y^2$ to give $\frac{\mathrm{d}(\pi y^2)}{\mathrm{d}y}=Ay\frac{\mathrm{d}y}{\mathrm{d}x}$ | M1 |
| | $4x\sin x=\pi y^2+2x \Rightarrow 4x\cos x+4\sin x=2\pi y\frac{\mathrm{d}y}{\mathrm{d}x}+2$<br>Fully correct differentiation. oe<br>Accept $4x\cos x\mathrm{d}x+4\sin x\mathrm{d}x=2\pi y\mathrm{d}y+2\mathrm{d}x$ | | A1 |
| | For the differentiation ignore any spurious "$\frac{\mathrm{d}y}{\mathrm{d}x}=$" | | |
| | **Alternative for first 3 marks using explicit differentiation:**<br>$y=\left(\frac{1}{\sqrt{\pi}}\right)(4x\sin x-2x)^{\frac{1}{2}}$ | | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\frac{1}{2\sqrt{\pi}}\right)(4x\sin x-2x)^{-\frac{1}{2}}(4x\cos x+4\sin x-2)$<br>M1: $\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x}=\pm 4x\cos x+4\sin x$ (as before)<br>M1: $(4x\sin x-2x)^{\frac{1}{2}}\to k(4x\sin x-2x)^{-\frac{1}{2}}$ | | M1 M1 |
| | Allow omission of π and sign errors when rearranging for the M marks | | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{\pi}}(4x\sin x-2x)^{-\frac{1}{2}}(4x\cos x+4\sin x-2)$ oe | | A1 |
| | $x=\frac{\pi}{2},\ y=1$<br>$\Rightarrow 4=2\pi\frac{\mathrm{d}y}{\mathrm{d}x}+2\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots\left(\frac{1}{\pi}\right)$ | Uses $x=\frac{\pi}{2}$ and $y=1$ to obtain a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (may be implied). For implicit differentiation, there must be a d$y$/d$x$ and there must be $x$'s and $y$'s. Explicit differentiation just requires use of $x=\frac{\pi}{2}$. | M1 |
| | $y-1="-\pi"\left(x-\frac{\pi}{2}\right)$ or $y="-\pi"x+c\Rightarrow c=1+\frac{\pi^2}{2}$<br>Uses normal gradient $-1\Big/\frac{\mathrm{d}y}{\mathrm{d}x}$ and $x=\frac{\pi}{2},\ y=1$ to find equation of normal.<br>Must use $-1\Big/\left(\text{their }\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ and $x=\frac{\pi}{2}$ and $y=1$ must be correctly placed.<br>If using $y = mx + c$ must reach as far as $c = \ldots$ | | M1 |
| | $y-1=-\pi\left(x-\frac{\pi}{2}\right)$ oe | Allow 3sf or more decimal equivalent answers e.g. $y=-3.14x+5.93$, $y-1=-3.14(x-1.57)$ etc. | A1cso |
| | | | **(6 marks)** |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **3(a)** | $(1+ax)^{-3}=1+(-3)(ax)+\frac{(-3)(-4)}{2!}(ax)^2+\frac{(-3)(-4)(-5)}{3!}(ax)^3+...$<br>Uses the binomial expansion with $n=-3$ and $'x'=ax$.<br>Minimum for M1 is $1+(-3)(ax)$ but can be scored for a correct 3rd or 4th term e.g. $\frac{(-3)(-4)}{2!}(ax)^2$ or $\frac{(-3)(-4)(-5)}{3!}(ax)^3$ | | M1 |
| | $=1-3ax+6a^2x^2-10a^3x^3+...$<br>or<br>$=1-3ax+6(ax)^2-10(ax)^3+...$ | A1: Three of the four terms correct and simplified<br>A1: All four terms correct and simplified and seen in part (a). | A1A1 |
| | | | **(3)** |
| **(b)** | $\text{f}(x)=\frac{2+3x}{(1+ax)^3}=(2+3x)(1-3ax+6a^2x^2-10a^3x^3)$<br>Writes $\text{f}(x)$ as $(2+3x)(1-3ax+6a^2x^2-10a^3x^3)$ using their expansion from part (a). This may be implied by their expansion. Do not condone 'invisible' brackets around 2 + 3$x$ or part(a) unless their presence is implied by later work and allow to recover in (b) from missing brackets in (a) e.g. $ax^2$ now becoming $a^2x^2$ | | M1 |
| | NB $\text{f}(x)=2+(3-6a)x+(12a^2-9a)x^2+(18a^2-20a^3)x^3$ | | |
| | $12a^2-9a=3$ | Multiplies out and sets their coefficient of $x^2$ (which comes from exactly 2 terms from their expansion – the two terms may have been combined earlier) = 3. | dM1 |
| | $4a^2-3a-1=(4a+1)(a-1)\Rightarrow a=...$<br>Correct method of solving a 3TQ. If working is shown see general guidance for correct methods. If no working is shown then you may need to check their values if their quadratic is incorrect. | | ddM1 |
| | $a=-\frac{1}{4}$ | Cao. Accept equivalent answers but must come from the **correct quadratic** and must be clearly identified. | A1 |
| | | | **(4)** |
| **(c)** | $18\left(-\frac{1}{4}\right)^2-20\left(-\frac{1}{4}\right)^3$ | Subs their $a=-\frac{1}{4}$ (positive or negative) into their coefficient of $x^3$ (which comes from exactly 2 terms from their expansion) | M1 |
| | Coefficient of $x^3$ is $\frac{23}{16}$ | Cao. Allow $\frac{23}{16}x^3$ | A1 |
| | | | **(2)** |
| | | | **9 marks** |

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>4 (a)</td><td colspan="2">$$\begin{array}{r} x^2+5 \\ x^2+x-12\overline{\big)\,x^4+x^3-7x^2+8x-48} \\ \underline{x^4+x^3-12x^2} \\ 5x^2+8x-48 \\ \underline{5x^2+5x-60} \\ 3x+12 \end{array}$$
M1: Divides $x^4+x^3-7x^2+8x-48$ by $x^2+x-12$ to get a quadratic quotient and a remainder of the form $\alpha x+\beta$ where $\alpha$ and $\beta$ are not both zero<br>A1: Correct quotient and remainder</td><td>M1A1</td></tr>
<tr><td></td><td colspan="2">$$\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv x^2+5+\frac{3(x+4) \text{ or } 3x+12}{(x+4)(x-3)}$$
Writes their answer as
$$\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv \text{Their Quotient}+\frac{\text{Their Remainder}}{(x+4)(x-3)}$$</td><td>M1</td></tr>
<tr><td></td><td colspan="2">$\equiv x^2+5+\dfrac{3}{(x-3)}$ or states $A$ = 5, $B$ = 3</td><td>A1</td></tr>
<tr><td></td><td colspan="2"></td><td>(4)</td></tr>
</table>

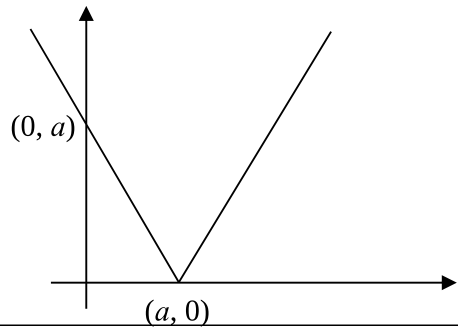
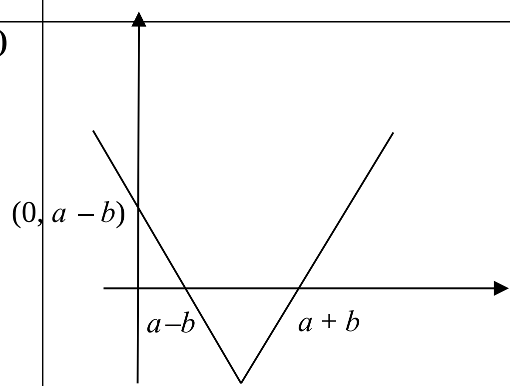
| | **Alternatives to part (a) by dividing by linear factors** | |
|---|---|---|
| | M1: Divides by $(x-3)$ first then divides by $(x+4)$ : $\left(x^4+x^3-7x^2+8x-48\right)\div(x-3):Q_1=x^3+4x^2+5x+23, R_1=21$ $\left(x^3+4x^2+5x+23\right)\div(x+4):Q_2=x^2+5, R_2=3$ For the M1, first division requires $Q_1$ to be a cubic and $R_1$ a constant and the second division to give a quadratic $Q_2$ and constant $R_2$ A1: Correct quotients and remainders | M1A1 |
| | $\dfrac{x^4+x^3-7x^2+8x-48}{(x+4)(x-3)}\equiv x^2+5+\dfrac{3}{x+4}+\dfrac{21}{(x-3)(x+4)}$ Writes their answer as $Q_2+\dfrac{R_2}{x+4}+\dfrac{R_1}{(x-3)(x+4)}$ | M1 |
| | $\equiv x^2+5+\dfrac{3}{(x-3)}$ or states $A=5$, $B=3$ | A1 |
| | M1: Divides by $(x+4)$ first then divides by $(x-3)$ : $\left(x^4+x^3-7x^2+8x-48\right)\div(x+4):Q_1=x^3-3x^2+5x-12, R_1=0$ $\left(x^3-3x^2+5x-12\right)\div(x-3):Q_2=x^2+5, R_2=3$ For the M1, first division requires $Q_1$ to be a cubic and $R_1$ a constant and the second division to give a quadratic $Q_2$ and constant $R_2$ A1: Correct quotients and remainders | M1A1 |
| | $\dfrac{x^4+x^3-7x^2+8x-48}{(x+4)(x-3)}\equiv x^2+5+\dfrac{3}{x-3}(+0)$ Writes their answer as $Q_2+\dfrac{R_2}{x-3}+\dfrac{R_1}{(x-3)(x+4)}$ | M1 |
| | $\equiv x^2+5+\dfrac{3}{(x-3)}$ or states $A=5$, $B=3$ | A1 |

| | | | |
|---|---|---|---|
| | **Alternative by comparing coefficients** | | |
| | $x^4+x^3-7x^2+8x-48 \equiv (x^2+A)(x^2+x-12)+B(x+4)$<br>Multiplies through by $(x^2+x-12)$ to obtain correct lhs and one of $(x^2+A)(x^2+x-12)$ or $B(x+4)$ on the rhs<br>If $(x^2+A)(x^2+x-12)$ is expanded, must see both $x^2(x^2+x-12)+A(x^2+x-12)$ | | M1 |
| | 2 correct equations<br>e.g. $x^2 \Rightarrow A-12=-7,\quad x \Rightarrow A+B=8,\quad \text{const} \Rightarrow -12A+4B=-48$ | | A1 |
| | $A=5,\ B=3$ | M1: Solves to obtain one of $A$ or $B$<br>A1: Both values correct | M1A1 |
| | **Alternative by substitution** | | |
| | $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv x^2+A+\dfrac{B}{x-3}$<br>$x=0 \Rightarrow 4=A-\dfrac{B}{3},\ x=1 \Rightarrow \dfrac{45}{10}=1+A-\dfrac{B}{2}$<br>M1: Substitutes 2 values for $x$ A1: 2 correct equations<br>**Multiplying through before substitution must satisfy the condition for multiplying through in the previous alternative.** | | M1A1 |
| | $A=5,\ B=3$ | M1: Solves to obtain one of $A$ or $B$<br>A1: Both values correct | M1A1 |

| | | | |
|---|---|---|---|
| **(b)** | $g'(x)=2x-\frac{3}{(x-3)^2}$ | M1: $x^2+A+\frac{B}{x-3}\to 2x\pm\frac{B}{(x-3)^2}$<br>A1: $x^2+A+\frac{B}{x-3}\to 2x-\frac{B}{(x-3)^2}$<br>Follow through their $B$ or the letter $B$ or a made up $B$. | M1A1ft |
| | **Special Case:**<br>If they write g($x$) as $x^2+5+\frac{3x+12}{(x-3)}$ and correctly attempt to differentiate as $2x$ + the quotient rule on $\frac{3x+12}{(x-3)}$ then the M mark is available but **not** the A1ft. It must be the correct quotient rule and the numerator must be a linear expression. | | |
| | $g'(4)=2\times4-\frac{3}{(4-3)^2}(=5)$ | Substitutes $x=4$ into their derivative | M1 |
| | Uses $m=g'(4)=(5)$ with $(4,\ g(4))=(4,24)$ to form eqn of tangent | | |
| | $y-24=5(x-4)$ | Correct method of finding an equation of the tangent. The gradient must be $g'(4)$ and the point must be an attempt on $(4,g(4))$ | M1 |
| | $y=5x+4$ | Cso. This mark may be withheld for an incorrect "$A$" earlier or any incorrect work leading to a correct gradient. | A1 |
| | | | **(5)** |
| | | | **(9 marks)** |
| | **Alternative to part (b) for first 3 marks** | | |
| | $g'(x)=\frac{(x^2+x-12)(4x^3+3x^2-14x+8)-(x^4+x^3-7x^2+8x-48)(2x+1)}{(x^2+x-12)^2}$<br>M1: Correct use of the quotient rule – there must be evidence of the application of $\frac{vu'-uv'}{v^2}$ or this formula quoted and attempted.<br>A1: Correct derivative | | M1A1 |
| | $g'(4)=\frac{8\times256-192\times9}{8^2}(=5)$ | Substitutes $x=4$ into their derivative | M1 |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| | **Note that $2^x$ can be replaced by $\mathrm{e}^{x\ln 2}$ throughout and allow omission of "d$x$" throughout** | | |
| **5** | $\int x2^x\,\mathrm{d}x = x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\,\mathrm{d}x$ | M1: Integrates by parts the right way around to obtain an expression of the form $ax2^x - \int b2^x\,\mathrm{d}x$.<br>Allow $a$ = 1 and/or $b$ = 1.<br>A1: $x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\,\mathrm{d}x$<br>(Does not need to be seen all on one line) | M1A1 |
| | $\int x2^x\,\mathrm{d}x = x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$ | dM1: Completes to obtain an expression of the form $...-k2^x$<br>A1: $x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$ | dM1A1 |
| | $\left[x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}\right]_0^2 = \left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - \left(\frac{0\times 2^0}{\ln 2} - \frac{2^0}{(\ln 2)^2}\right)$<br>Uses the limits 0 and 2 and subtracts the right way round.<br>F(0) may be implied by e.g. $\frac{1}{(\ln 2)^2}$<br>But $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - (0)$ or just $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right)$ is ddM0 | | ddM1 |
| | $\left(= \frac{8}{\ln 2} - \frac{4}{(\ln 2)^2} + \frac{1}{(\ln 2)^2}\right)$ | | |
| | $= \frac{8\ln 2 - 3}{(\ln 2)^2}$ | Correct simplified fraction.<br>Allow equivalent simplified forms<br>e.g. $\frac{\ln 256 - 3}{(\ln 2)^2}, \frac{\ln 2^8 - 3}{(\ln 2)^2}$<br>Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$ | A1 |
| | | | **(6 marks)** |

| | | | |
|---|---|---|---|
| | **Alternative by substitution:** | | |
| | $u=2^x \Rightarrow \int x2^x\mathrm{d}x=\int \frac{\ln u}{\ln 2}.u.\frac{1}{u\ln 2}\mathrm{d}u=\int \frac{\ln u}{(\ln 2)^2}\ \mathrm{d}u$ | | |
| | $\int \frac{\ln u}{(\ln 2)^2}\ \mathrm{d}u=\frac{1}{(\ln 2)^2}\left(u\ln u-\int\ \mathrm{d}u\right)$ | M1: Integrates by parts the right way around to obtain an expression of the form $au\ln u-\int b\mathrm{d}u$.<br>Allow $a$ = 1 and/or $b$ = 1.<br>A1: $\frac{1}{(\ln 2)^2}\left(u\ln u-\int\ \mathrm{d}u\right)$ | M1A1 |
| | $\int \frac{\ln u}{(\ln 2)^2}\ \mathrm{d}u=\frac{1}{(\ln 2)^2}(u\ln u-u)$ | dM1: Completes to obtain an expression of the form $...-ku$<br>A1: $\frac{1}{(\ln 2)^2}(u\ln u-u)$ | dM1A1 |
| | $\left[\frac{1}{(\ln 2)^2}(u\ln u-u)\right]_1^4=\frac{1}{(\ln 2)^2}(4\ln 4-4)-(\ln 1-1)$<br>Uses the limits 1 and 4 and subtracts the right way round. | | M1 |
| | $=\frac{4\ln 4-3}{(\ln 2)^2}$ | Correct simplified fraction.<br>Allow equivalent simplified forms<br>e.g. $\frac{\ln 256-3}{(\ln 2)^2},\frac{\ln 2^8-3}{(\ln 2)^2},$<br>Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$ | A1 |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **6(a)(i)** |  | V shape with vertex on *x*-axis but **not** at the origin. | B1 |
| | | Correct V shape with (0, *a*) or just *a* and (*a*, 0) or just *a* marked in the correct places. Left branch must cross or touch the *y*-axis. Allow coordinates the wrong way round if marked in the correct place. | B1 |
| | | | **(2)** |
| **(a)(ii)** |  | Their part (i) translated down (by any amount) but clearly not left or right, or the correct shape i.e. a V with the vertex in 4th quadrant. | B1ft |
| | | A *y*-intercept of *a* – *b* on the positive *y*-axis or intercepts of *a* – *b* and *a* + *b* on the positive *x*-axis with *a* + *b* to the right of *a* – *b* | B1 |
| | | A fully correct diagram. | B1 |
| | | | **(3)** |
| **(b)** | $x-a-b=\frac{1}{2}x \Rightarrow x=\ldots$<br>**or**<br>$-x+a-b=\frac{1}{2}x \Rightarrow x=\ldots$ | Solves $x-a-b=\frac{1}{2}x$ **or** solves $-x+a-b=\frac{1}{2}x$ as far as $x=\ldots$<br>Allow $<$ or $>$ for $=$. | M1 |
| | $x-a-b=\frac{1}{2}x \Rightarrow x=\ldots$<br>**and**<br>$-x+a-b=\frac{1}{2}x \Rightarrow x=\ldots$ | Solves $x-a-b=\frac{1}{2}x$ **and** solves $-x+a-b=\frac{1}{2}x$ as far as $x=\ldots$<br>Allow $<$ or $>$ for $=$. | M1 |
| | $\frac{2}{3}(a-b)<x<2(a+b)$ | ddM1: Chooses inside region.<br>A1: Allow alternatives e.g.<br>$x<2(a+b)$ **and** $x>\frac{2}{3}(a-b)$,<br>$x<2(a+b) \cap x>\frac{2}{3}(a-b)$,<br>$\left(\frac{2}{3}(a-b),\ 2(a+b)\right)$ but not<br>$x<2(a+b)$**,** $x>\frac{2}{3}(a-b)$ | ddM1A1 |
| | | | **(4)** |
| | | | **(9 marks)** |

| | | | |
|---|---|---|---|
| | **Attempts at squaring in (b)** | | |
| | $(x-a)^2=\left(\frac{1}{2}x+b\right)^2$ | | |
| | $(x-a)^2=\left(\frac{1}{2}x+b\right)^2 \Rightarrow 3x^2-4x(2a+b)+4\left(a^2-b^2\right)=0$<br>Squares both sides and obtains 3TQ = 0 | | M1 |
| | $x=\frac{4(2a+b)\pm 4(a+2b)}{6}$<br>$\left(=2(a+b),\ \frac{2}{3}(a-b)\right)$ | Attempt to solve 3TQ applying usual rules | M1 |
| | $\frac{2}{3}(a-b)<x<2(a+b)$ | ddM1: Chooses inside region. **Dependent on both previous M marks.**<br>A1: Allow alternatives e.g.<br>$x<2(a+b)$ **and** $x>\frac{2}{3}(a-b)$, $\left(\frac{2}{3}(a-b),\ 2(a+b)\right)$ but not $x<2(a+b)$**,** $x>\frac{2}{3}(a-b)$<br>Expressions must have just one term in *a* and one term in *b*. | ddM1A1 |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **7 (a)** | Strip width = 1 | May be implied by their trapezium rule. | B1 |
| | Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{15}}+2\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)\right)$ <br> $\approx \frac{1}{2}(0.33...+0.25...+2(0.30...+0.27...))$ | M1: Correct structure for the $y$ values. Look for ($y$ at $x$ = 2) + ($y$ at $x$ = 5) + 2(sum of other $y$ values). <br> A1: Correct numerical expression. If decimals are used, look for awrt 1dp initially, however a correct final answer would imply this mark. | M1 A1 |
| | Awrt 0.875 | | A1 |
| | | | **(4)** |
| | May use separate trapezia: | | |
| | Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{11}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{15}}\right)$ | | |
| | B1: Strip width = 1 <br> M1: Correct structure for the $y$ values as above <br> A1: Correct expression as described above <br> A1: Awrt 0.875 | | |
| **(b)** | $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x=(2x+5)^{\frac{1}{2}}$ | M1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x=k(2x+5)^{\frac{1}{2}}$ <br> A1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x=(2x+5)^{\frac{1}{2}}$ | M1A1 |
| | $\int_2^5 \frac{1}{\sqrt{2x+5}}\mathrm{d}x=(2(5)+5)^{\frac{1}{2}}-(2(2)+5)^{\frac{1}{2}}$ | Substitutes 5 and 2 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… $-3$ unless the substitution of 5 and 2 is explicitly seen. | dM1 |
| | $=\sqrt{15}-\sqrt{9}\ (=\sqrt{15}-3)$ | $\sqrt{15}-\sqrt{9}$ or $\sqrt{15}-3$ | A1 |
| | | | **(4)** |

| | | | |
|---|---|---|---|
| | **Alternative to (b) by substitution** $u=2x+5$ | | |
| | $u=2x+5\Rightarrow\int\frac{1}{\sqrt{2x+5}}\,\mathrm{d}x=\int\frac{1}{\sqrt{u}}\frac{1}{2}\,\mathrm{d}u$ | M1: $\int\frac{1}{\sqrt{2x+5}}\,\mathrm{d}x=ku^{\frac{1}{2}}$<br>A1: $\int\frac{1}{\sqrt{2x+5}}\,\mathrm{d}x=u^{\frac{1}{2}}$ | M1A1 |
| | $\int_2^5\frac{1}{\sqrt{2x+5}}\,\mathrm{d}x=(15)^{\frac{1}{2}}-(9)^{\frac{1}{2}}$ | Substitutes 15 and 9 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… $-3$ unless the substitution of 15 and 9 is explicitly seen. | dM1 |
| | $=\sqrt{15}-\sqrt{9}\left(=\sqrt{15}-3\right)$ | $\sqrt{15}-\sqrt{9}$ or $\sqrt{15}-3$ | A1 |
| | **Alternative to (b) by substitution** $u=(2x+5)^{\frac{1}{2}}$ | | |
| | $u=(2x+5)^{\frac{1}{2}}\Rightarrow\int\frac{1}{u}.u\,\mathrm{d}u=\int u\,\mathrm{d}u$ | M1: $\int\frac{1}{\sqrt{2x+5}}\,\mathrm{d}x=ku$<br>A1: $\int\frac{1}{\sqrt{2x+5}}\,\mathrm{d}x=u$ | M1A1 |
| | $\int_2^5\frac{1}{\sqrt{2x+5}}\,\mathrm{d}x=(15)^{\frac{1}{2}}-(9)^{\frac{1}{2}}$ | Substitutes √15 and 3 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… $-3$ unless the substitution of √15 and 3 is explicitly seen. | dM1 |
| | $=\sqrt{15}-\sqrt{9}\left(=\sqrt{15}-3\right)$ | $\sqrt{15}-\sqrt{9}$ or $\sqrt{15}-3$ | A1 |
| **(c)** | $\pm(\text{correct}(a)-\text{correct}(b))=\pm0.002$<br>or<br>$\pm\frac{\text{correct}(a)-\text{correct}(b)}{\text{correct}(b)}\times100=\pm0.2\%$ | Finds the magnitude of the error and writes as ±0.002 or $\pm2\times10^{-3}$ or ±0.2%<br>Or finds the percentage error and writes as ±0.2% | B1 |
| | | | **(1)** |
| | | | **(9 marks)** |

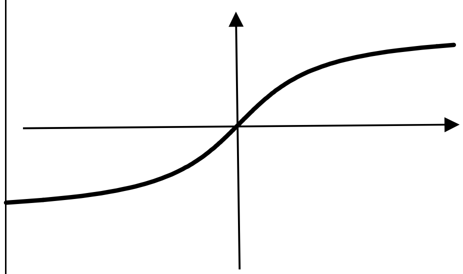
| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8 (a)** | $\sin 2x - \tan x \equiv 2\sin x\cos x - \dfrac{\sin x}{\cos x}$ | Uses a **correct** identity for sin2$x$ | M1 |
| | $\equiv \dfrac{2\sin x\cos x\cos x}{\cos x} - \dfrac{\sin x}{\cos x}$ | Obtains common denominator. This is **NOT** dependent upon the previous M so accept expressions like, $\sin 2x - \tan x \equiv \sin 2x - \dfrac{\sin x}{\cos x}$ $= \dfrac{\sin 2x\cos x - \sin x}{\cos x}$ | M1 |
| | $\equiv \dfrac{2\cos^2 x\sin x - \sin x}{\cos x}$ | Correct fraction with just sin $x$ and cos $x$ | A1 |
| | $\equiv \dfrac{(2\cos^2 x - 1)\sin x}{\cos x} \equiv \cos 2x\tan x$* | Uses a correct identity for cos2$x$ and completes correctly with no errors. An error could be for example, mixed variables used or loss of an $x$ along the way. | A1* |
| | | | **(4)** |
| | **Alternative 1 for (a)** | | |
| | $\sin 2x - \tan x \equiv 2\sin x\cos x - \dfrac{\sin x}{\cos x}$ | Uses a **correct** identity for sin2$x$ | M1 |
| | $\dfrac{\sin x}{\cos x}\left(2\cos^2 x - 1\right)$ | M1: Takes out a factor of $\dfrac{\sin x}{\cos x}$<br>A1: Correct expression | M1A1 |
| | $\equiv \tan x\cos 2x$* | Completes correctly with no errors. | A1* |
| | | | |
| | **Alternative 2 for (a)** | | |
| | $2\sin x\cos x - \dfrac{\sin x}{\cos x} \equiv \dfrac{\sin x}{\cos x}\left(\cos^2 x - \sin^2 x\right)$ | Uses a **correct** identity for sin2$x$ | M1 |
| | $2\sin x\cos^2 x - \sin x \equiv \sin x\left(\cos^2 x - \sin^2 x\right)$ | Multiplies **both sides** by cos $x$ | M1 |
| | $2\cos^2 x - 1 \equiv \left(\cos^2 x - \sin^2 x\right)$ | Correct identity | A1 |
| | This is true* | Conclusion provided | A1* |
| | | | |
| | **Alternative 3 for (a)** | | |
| | $\tan x\cos 2x \equiv \dfrac{\sin x}{\cos x}\left(2\cos^2 x - 1\right)$ | Uses a **correct** identity for cos2$x$ | M1 |
| | $\equiv 2\sin x\cos x - \dfrac{\sin x}{\cos x}$ | M1: Multiplies out<br>A1: Correct expression | M1A1 |
| | $\equiv \sin 2x - \tan x$* | A1: Obtains lhs with no errors | A1* |

| | | | |
|---|---|---|---|
| **8(b)(i)** | $\sin 2\theta - \tan\theta = \sqrt{3}\cos 2\theta \Rightarrow \tan\theta\cos 2\theta = \sqrt{3}\cos 2\theta$ | | |
| | $\tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} = (\text{awrt } 1.05)$ | M1: $\tan\theta = \pm\sqrt{3} \Rightarrow \theta = ...$<br>A1: $\theta = \frac{\pi}{3}$ Accept awrt 1.05. Ignore solutions outside the range but withhold the A mark for extra solutions in range. | M1A1 |
| | $\cos 2\theta = 0 \Rightarrow \theta = \frac{\pi}{4}$ (awrt 0.785) | M1: $\cos 2\theta = 0 \Rightarrow \theta = ...$<br>A1: $\theta = \frac{\pi}{4}$ Accept awrt 0.785. Ignore solutions outside the range but withhold the A mark for extra solutions in range. | M1A1 |
| | | | |
| **(b)(ii)** | $\tan(\theta+1)\cos(2\theta+2) - \sin(2\theta+2) = 2 \Rightarrow \tan(\theta+1) = -2$<br>M1: $\tan(\theta+1) = \pm 2$ | | M1 |
| | $\Rightarrow \theta = \arctan(-2) - 1$ | Correct order of operations i.e. $\theta = \arctan(\pm 2) - 1$. This may be implied by $\theta = -2.1\ldots$ | dM1 |
| | $\Rightarrow \theta = 1.03$ | awrt $\theta = 1.03$. Ignore solutions outside the range but withhold the A mark for extra solutions in range. | A1 |
| | | | |
| | | | **(7)** |
| | | | **(11 marks)** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **9.(a)** | $t = 0 \Rightarrow P = \dfrac{9000}{3+7} = 900$ | M1: Sets $t = 0$, may be implied by $e^0 = 1$ or may be implied by $\dfrac{9000}{3+7}$ or by a correct answer of 900. | M1A1 |
| | | A1: 900 | |
| | | | **(2)** |
| **(b)** | $t \to \infty \quad P \to \dfrac{9000}{3} = 3000$ | Sight of 3000 | B1 |
| | | | **(1)** |
| **(c)** | $t = 4,\ P = 2500 \Rightarrow 2500 = \dfrac{9000e^{4k}}{3e^{4k}+7}$ | Correct equation with $t = 4$ and $P = 2500$ | B1 |
| | $e^{4k} = \dfrac{17500}{1500} = (\text{awrt } 11.7 \text{ or } 11.6)$ or $e^{-4k} = \dfrac{1500}{17500} = (\text{awrt } 0.857)$ | M1: Rearranges the equation to make $e^{\pm 4k}$ the subject. They need to multiply by the $3e^{4k} + 7$ term, and collect terms in $e^{4k}$ or $e^{-4k}$ reaching $e^{\pm 4k} = C$ where C is a constant. | M1A1 |
| | | A1: Achieves intermediate answer of $e^{4k} = \dfrac{17500}{1500} = (\text{awrt } 11.7 \text{ or } 11.6)$ or $e^{-4k} = \dfrac{1500}{17500} = (\text{awrt } 0.857)$ | |
| | $k = \dfrac{1}{4}\ln\left(\dfrac{35}{3}\right)$ **or** awrt 0.614 | **d**M1: Proceeds from $e^{\pm 4k} = C,\quad C > 0$ by correctly taking ln's and then making $k$ the subject of the formula. Award for e.g. $e^{4k} = C \Rightarrow 4k = \ln(C) \Rightarrow k = \dfrac{\ln(C)}{4}$ | **d**M1A1 |
| | | A1: cao: Awrt 0.614 or the correct exact answer (or equivalent) | |
| | | | **(5)** |

| | | | |
|---|---|---|---|
| | Alternative correct work in (c): | | |
| | $t = 4,\ P = 2500 \Rightarrow 2500 = \dfrac{9000e^{4k}}{3e^{4k}+7}$ | Correct equation with $t = 4$ and $P = 2500$ | B1 |
| | $7500e^{4k} + 17500 = 9000e^{4k}$ | | |
| | $1500e^{4k} = 17500$ | | |
| | $\ln 1500 + \ln e^{4k} = \ln 17500$ | M1: Takes ln's correctly<br>A1: Correct equation | M1A1 |
| | $\ln e^{4k} = \ln 17500 - \ln 1500$ | | |
| | $4k = \ln 17500 - \ln 1500$ | | |
| | $k = \dfrac{\ln 17500 - \ln 1500}{4}$ | Makes $k$ the subject | M1A1 |
| | $k = \dfrac{1}{4}\ln\left(\dfrac{35}{3}\right)$ **or** awrt 0.614 | cao: Awrt 0.614 or the correct exact answer (or equivalent) | |

| | | | |
|---|---|---|---|
| **(d)** | $\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{(3e^{kt}+7)\times 9000ke^{kt} - 9000e^{kt}\times 3ke^{kt}}{(3e^{kt}+7)^2}\left(=\frac{63000ke^{kt}}{(3e^{kt}+7)^2}\right)$<br>Differentiates using the quotient rule to achieve<br>$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{(3e^{kt}+7)\times Pe^{kt} - 9000e^{kt}\times Qe^{kt}}{(3e^{kt}+7)^2}$<br>**or**<br>$\frac{\mathrm{d}P}{\mathrm{d}t} = 9000k\mathrm{e}^{kt}\left(3\mathrm{e}^{kt}+7\right)^{-1} - 9000\mathrm{e}^{kt}\left(3\mathrm{e}^{kt}+7\right)^{-2}\times 3k\mathrm{e}^{kt}$<br>Differentiates using the product rule to achieve<br>$\frac{\mathrm{d}P}{\mathrm{d}t} = P\mathrm{e}^{kt}\left(3\mathrm{e}^{kt}+7\right)^{-1} - 9000\mathrm{e}^{kt}\left(3\mathrm{e}^{kt}+7\right)^{-2}\times Q\mathrm{e}^{kt}$<br>**or**<br>$\frac{\mathrm{d}P}{\mathrm{d}t} = 63000k\mathrm{e}^{-kt}\left(3+7\mathrm{e}^{-kt}\right)^{-2}$<br>Differentiates using the chain rule on $P = 9000\left(3+7\mathrm{e}^{-kt}\right)^{-1}$ to achieve<br>$\frac{\mathrm{d}P}{\mathrm{d}t} = \pm D\mathrm{e}^{-kt}\left(3+7\mathrm{e}^{-kt}\right)^{-2}$<br>**Watch for** $\mathrm{e}^{kt} \to kt\mathrm{e}^{kt}$ **which is M0** | | M1 |
| | Sub $t$ =10 and $k = 0.614 \Rightarrow \frac{\mathrm{d}P}{\mathrm{d}t} = ...$ | Substitutes $t$ = 10 and their $k$ to obtain a value for $\frac{\mathrm{d}P}{\mathrm{d}t}$. If the value for $\frac{\mathrm{d}P}{\mathrm{d}t}$ is incorrect then the **substitution** of $t$ = 10 must be seen explicitly. | dM1<br>(A1 on Epen) |
| | $\frac{\mathrm{d}P}{\mathrm{d}t} = 9$ | Awrt 9 (NB $\frac{\mathrm{d}P}{\mathrm{d}t} = 9.1694...$) | A1 |
| | | | **(3)** |
| | | | **(11 marks)** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **10(a)** |  | M1: Curve not a straight line through (0, 0) in quadrants 1 and 3 only.<br>A1: Grad $\to 0$ as $x \to \pm\infty$ | M1A1 |
| | | | **(2)** |
| **(b)** | $3\arctan(x+1)-\pi=0$<br>$\Rightarrow \arctan(x+1)=\dfrac{\pi}{3}$ | Substitutes $g(x+1)=\arctan(x+1)$ in $3g(x+1)-\pi=0$ and makes $\arctan(x+1)$ the subject. Do not condone missing brackets unless later work implies their presence. | M1 |
| | $\Rightarrow x=\tan\left(\dfrac{\pi}{3}\right)-1=\sqrt{3}-1$ | dM1: Takes tan and makes $x$ the subject e.g. allow $x=\sqrt{3}\pm 1$. Note that $\tan\left(\dfrac{\pi}{3}\right)$ does not need to be evaluated for this mark. May be implied by e.g. $x=0.732\ldots$<br>A1: $\sqrt{3}-1$ | dM1A1 |
| | | | **(3)** |
| **(c)** | Sub $x=5$ **and** $x=6$ into $\pm\left(\arctan x-4+\dfrac{1}{2}x\right) \Rightarrow -0.126..,+0.405...$<br>**and obtains at least one answer correct to 1sf** | | M1 |
| | Both values correct (to one sig fig), change of sign + conclusion<br>Allow equivalent statements e.g. positive, negative therefore root etc. but this mark may be withheld if there are any contradictory statements e.g. therefore root lies between g(5) and g(6) | | A1 |
| | If $-\left(\arctan x-4+\dfrac{1}{2}x\right)$ is used to give $0.126..,-0.405...$, allow both marks if a conclusion is given. | | |
| | | | **(2)** |
| **(d)** | $x_1=8-2\arctan 5$ | Score for $x_1=8-2\arctan 5=\ldots$<br>This may be implied by awrt 5.3 (radians) or awrt -149 (degrees) for $x_1$ | M1 |
| | $x_1=5.253,\quad x_2=5.235$ | $x_1=$ awrt $5.253,\quad x_2=$ awrt $5.235$<br>Ignore any subsequent iterations and ignore labelling if answers are clearly the second and third terms. | A1 |
| | | | **(2)** |
| | | | **(9 marks)** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **11 (a)** | $\begin{pmatrix}7\\4\\9\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\4\end{pmatrix}=\begin{pmatrix}-6\\-7\\3\end{pmatrix}+\mu\begin{pmatrix}5\\4\\b\end{pmatrix}\Rightarrow \begin{matrix}7+1\lambda=-6+5\mu\\4+1\lambda=-7+4\mu\\9+4\lambda=3+b\mu\end{matrix}$ any two of<br>Writes down any two equations for the coordinates of the point of intersection. There must be an attempt to set the coordinates equal but condone slips. | | M1 |
| | Full method to find both $\lambda$ and $\mu$ from equations 1 and 2 and uses these values and equation 3 to find a value for $b$ | | **d**M1 |
| | $(1)-(2)\Rightarrow 3=1+\mu\Rightarrow\mu=2$ | | |
| | Sub $\mu=2$ into (1) $\Rightarrow 7+1\lambda=-6+10\Rightarrow\lambda=-3$ | | |
| | Put values in 3rd equation $9-12=3+2b\Rightarrow b=-3$*<br>Completely correct work including $\lambda=-3,\ \ \mu=2$ and substitution into **both** sides of the third equation to give $b=-3$ | | A1 |
| | Position vector of intersection is $\begin{pmatrix}7\\4\\9\end{pmatrix}+-3\begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\begin{pmatrix}-6\\-7\\3\end{pmatrix}+2\begin{pmatrix}5\\4\\-3\end{pmatrix}$<br>Substitutes their value of $\lambda$ into $l_1$ to find the coordinates or position vector of the point of intersection. Alternatively substitutes their value of $\mu$ into $l_2$ to find the coordinates or position vector of the point of intersection.<br>**May be implied by at least 2 correct coordinates for X** | | **d**M1 |
| | $X=(4,\ 1,\ -3)$ | Correct coordinates or vector. Correct coordinates implies M1A1 Marks for finding the coordinates of $X$ can score anywhere in the question. | A1 |
| | | | **(5)** |
| | **(b) Way 1** | | |
| **(b)** | $\pm\overrightarrow{XA}=\pm\begin{pmatrix}2\\2\\8\end{pmatrix},\quad \pm\overrightarrow{XB}=\pm\begin{pmatrix}10\\8\\-6\end{pmatrix}$ | Attempts the difference between the coordinates $X$ and $A$, $X$ and $B$. This could be implied by the calculation of the lengths $AX$ and $BX$. Allow slips but must be subtracting. | M1 |
| | $\pm\overrightarrow{XA}.\pm\overrightarrow{XB}=\lvert XA\rvert\lvert XB\rvert\cos\theta\Rightarrow 20+16-48=\sqrt{72}\sqrt{200}\cos\theta$<br>M1: Attempt the scalar product of $\overrightarrow{XA}$ and $\overrightarrow{XB}$ or $\overrightarrow{AX}$ and $\overrightarrow{BX}$ or $\overrightarrow{XA}$ and $\overrightarrow{BX}$ or $\overrightarrow{AX}$ and $\overrightarrow{XB}$<br>Allow $\cos\theta=\dfrac{\begin{pmatrix}2\\2\\8\end{pmatrix}\bullet\begin{pmatrix}10\\8\\-6\end{pmatrix}}{\sqrt{72}\sqrt{200}}$ for M1 but not A1 unless the numerator is evaluated<br>A1: A correct un-simplified expression $20+16-48=\sqrt{72}\sqrt{200}\cos\theta$ oe | | dM1A1 |
| | $\cos\theta=\dfrac{-12}{\sqrt{72}\times\sqrt{200}}\Rightarrow\theta=\arccos\left(-\dfrac{1}{10}\right)$* | This is a given answer. There must be an intermediate line with $\cos\theta=$.. or $\theta=$ … | A1* |
| | | | **(4)** |

| | (b) Way 2 | | |
|---|---|---|---|
| **(b)** | $\mathbf{d}_1 = \begin{pmatrix}1\\1\\4\end{pmatrix},\quad \mathbf{d}_2 = \begin{pmatrix}5\\4\\-3\end{pmatrix}$ | Uses $b$ = -3 and the direction vectors **or multiples of the direction vectors** | M1 |
| | $\mathbf{d}_1.\mathbf{d}_2 = \lvert\mathbf{d}_1\rvert\lvert\mathbf{d}_2\rvert\cos\theta \Rightarrow 5+4-12=\sqrt{18}\sqrt{50}\cos\theta$<br>M1: Attempt the scalar product of the direction vectors<br>Allow $\cos\theta = \dfrac{\begin{pmatrix}1\\1\\4\end{pmatrix}\bullet\begin{pmatrix}5\\4\\-3\end{pmatrix}}{\sqrt{18}\sqrt{50}}$ for M1 but not A1 unless the numerator is evaluated<br>A1: A correct un-simplified expression $5+4-12=\sqrt{18}\sqrt{50}\cos\theta$ oe | | dM1A1 |
| | $\cos\theta = \dfrac{-3}{\sqrt{18}\times\sqrt{50}} \Rightarrow \theta = \arccos\left(-\dfrac{1}{10}\right)*$ | This is a given answer. There must be an intermediate line with $\cos\theta$=.. or $\theta$= … | A1* |

| | (b) Way 3 | | |
|---|---|---|---|
| **(b)** | $\pm\overrightarrow{XA} = \pm\begin{pmatrix}2\\2\\8\end{pmatrix},\quad \pm\overrightarrow{XB} = \pm\begin{pmatrix}10\\8\\-6\end{pmatrix}$ | Attempts the difference between the coordinates $X$ and $A$, $X$ and $B$. This could be implied by the calculation of the lengths $AX$ and $BX$. Allow slips but must be subtracting. | M1 |
| | $\lvert AB\rvert^2 = \lvert XA\rvert^2+\lvert XB\rvert^2-2\lvert XA\rvert\lvert XB\rvert\cos\theta \Rightarrow 8^2+6^2+14^2 = 72+200-2\sqrt{72}\sqrt{200}\cos\theta$<br>M1: Uses $\overrightarrow{AB}$ with a correct attempt at the cosine rule<br>A1: A correct un-simplified expression $8^2+6^2+14^2 = 72+200-2\sqrt{72}\sqrt{200}\cos\theta$ oe | | dM1A1 |
| | $\cos\theta = \dfrac{-24}{2\sqrt{72}\times\sqrt{200}} \Rightarrow \theta = \arccos\left(-\dfrac{1}{10}\right)*$ | This is a given answer. There must be an intermediate line with $\cos\theta$=.. or $\theta$= … | A1* |
| **(c)** | $\cos\theta = -\dfrac{1}{10} \Rightarrow \sin\theta = \dfrac{\sqrt{99}}{10}$ | oe e.g. $\sqrt{\dfrac{99}{100}}, \dfrac{3\sqrt{11}}{10}$. May be implied by a correct exact area. | B1 |
| | Area of triangle = $\frac{1}{2}XA\times XB\times\sin\theta$ $\quad A=\frac{1}{2}\times6\sqrt{2}\times10\sqrt{2}\times\frac{3\sqrt{11}}{10}$<br>Uses Area of triangle = $\frac{1}{2}XA\times XB\times\sin\theta$<br>This mark can be scored for e.g. $\frac{1}{2}(\text{their } XA)\times(\text{their } XB)\times\sin\left(\cos^{-1}\left(-\frac{1}{10}\right)\right)$ or<br>$\frac{1}{2}(\text{their } XA)\times(\text{their } XB)\times\sin(95.7391...)$<br>Must be using the angle given by $\cos^{-1}\left(-\frac{1}{10}\right)$ | | M1 |
| | $A=18\sqrt{11}$ oe | Accept for example $A=9\sqrt{44}, \sqrt{3564}$ | A1 |
| | **Note that** $A=\frac{1}{2}\times6\sqrt{2}\times10\sqrt{2}\times\sin(95.7391...)=18\sqrt{11}$ **scores all 3 marks** | | |
| | | | **(3)** |
| | | | **(12 marks)** |

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| **12.(a)** | $V = \int y^2 \mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t = \int (2\sin 2t)^2 3\cos t\,\mathrm{d}t$<br>M1: Attempts $\int y^2 \mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$ where $\frac{\mathrm{d}x}{\mathrm{d}t} = \pm k\cos t$<br>May be implied by e.g. $\int (2\sin 2t)^2 3\cos t$<br>A1: $= \int (2\sin 2t)^2 3\cos t\,(\mathrm{d}t)$ (d$t$ can be missing as long as the M is scored) | | M1A1 |
| | $= \int (4\sin t\cos t)^2 3\cos t\,\mathrm{d}t$ | Uses sin2$t$ = 2sin$t$cos$t$ | M1 |
| | $x = \frac{3}{2} \Rightarrow t = \frac{\pi}{6}$ or $k = 48$ | Correct value for $a$ (must be exact) or a correct value for $k$ | B1 |
| | $V = \int \pi y^2 \mathrm{d}x = 48\pi \int_0^{\frac{\pi}{6}} \sin^2 t\cos^3 t\,\mathrm{d}t$ * | Achieves printed answer including "d$t$" (even if lost earlier) with correct limits and 48$\pi$ in place with no errors. Or achieves the printed answer with the letters $a$ and $k$ and states the correct values of $a$ and $k$. | A1* |
| | | | **(5)** |

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| **(b)** | $u=\sin t \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}t}=\cos t$ | States $\dfrac{\mathrm{d}u}{\mathrm{d}t}=\cos t$ or equivalent. May be implied. | B1 |
| | $V=k\int \sin^2 t\cos^3 t\,\mathrm{d}t=k\int u^2\cos^2 t\,\mathrm{d}u=k\int u^2(1-\sin^2 t)\,\mathrm{d}u=k\int u^2(1-u^2)\,\mathrm{d}u$<br>M1: Substitutes **fully** including for d$t$ using $u$ = sin $t$ and $\cos^2 t=\pm1\pm\sin^2 t$ to produce an integral just in terms of $u$.<br>A1ft: Fully correct integral in terms of $u$ - follow through on incorrect $k$'s and ignore inclusion or omission of $\pi$ so look for e.g. $k\int u^2(1-u^2)\,\mathrm{d}u$ or equivalent and allow the letter $k$. | | M1A1ft |
| | $=k\left[\dfrac{u^3}{3}-\dfrac{u^5}{5}\right]$ | Multiplies out to form a polynomial in $u$ and integrates with $u^n\to u^{n+1}$ for at least one of their powers of $u$. | M1 |
| | Volume = $48\pi\left[\dfrac{u^3}{3}-\dfrac{u^5}{5}\right]_0^{\frac{1}{2}}=\dfrac{17\pi}{10}$ | **d**M1: All methods must have been scored. It is for using the limits 0 and $\dfrac{1}{2}$ and subtracting or for using the limits 0 and $\frac{\pi}{6}$ if they return to sin $t$. However, in both cases the substitution of 0 does not need not be seen. | **d**M1A1 |
| | | A1: $V=\dfrac{17\pi}{10}$ oe such as $V=\dfrac{51\pi}{30}$ | |
| | | | **(6)** |
| | If $\dfrac{\mathrm{d}u}{\mathrm{d}t}=-\cos t$ is used, maximum B0M1A0M1M1A0 is possible | | |
| | | | **(11 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **13(a)** | $V=\frac{1}{3}\pi h^2(30-h)=10\pi h^2-\frac{1}{3}\pi h^3 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h}=20\pi h-\pi h^2$<br>or<br>$V=\frac{1}{3}\pi h^2(30-h)\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h}=\frac{2}{3}\pi h(30-h)-\frac{1}{3}\pi h^2$ | M1A1 |
| | **M1:** Attempts $\frac{\mathrm{d}V}{\mathrm{d}h}$ either by multiplying out and differentiating each term to give a derivative of the form $\alpha h-\beta h^2$ or by the product rule to give a derivative of the form $\alpha h(30-h)\pm\beta h^2$.<br>**A1:** Any correct (possibly un-simplified) form for $\frac{\mathrm{d}V}{\mathrm{d}h}$ | |
| | Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow -\frac{1}{10}V=\left(20\pi h-\pi h^2\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}$ | M1 |
| | Uses a **correct** form of the chain rule, e.g. $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ or uses $\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{1}{10}V$. | |
| | $\Rightarrow -\frac{1}{10}\times\frac{1}{3}\pi h^2(30-h)=\pi h(20-h)\times\frac{\mathrm{d}h}{\mathrm{d}t}\left(\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=...\right)$ | M1 |
| | Substitutes $V=\frac{1}{3}\pi h^2(30-h)$ and rearranges to obtain $\frac{\mathrm{d}h}{\mathrm{d}t}$ in terms of $h$ | |
| | $\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{h(30-h)}{30(20-h)}$ * — This is a given answer. There must have been intermediate lines and correct factorisation and no errors and "$\frac{\mathrm{d}h}{\mathrm{d}t}=$" must be seen at some point. | A1* |
| | | **(5)** |
| **(b)** | $\frac{30(20-h)}{h(30-h)}\equiv\frac{A}{h}+\frac{B}{30-h}$ — Correct form for the partial fractions | B1 |
| | $30(20-h)\equiv A(30-h)+Bh$<br>$h=30\Rightarrow 30B=-300\Rightarrow B=-10$ and $h=0\Rightarrow 30A=600\Rightarrow A=20$<br><br>Attempts to get both constants by a correct method e.g. substituting, comparing coefficients, cover up rule | M1 |
| | $\frac{30(20-h)}{h(30-h)}\equiv\frac{20}{h}-\frac{10}{30-h}$ — Correct partial fractions (or states "$A$" = 20, "$B$" = $-10$) | A1 |
| | | **(3)** |

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| **(c)** | **Way 1** | | |
| | $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h(30-h)}{30(20-h)} \Rightarrow \int \frac{30(20-h)}{h(30-h)}\,\mathrm{d}h = -1\int \mathrm{d}t$<br>A correct statement which may be implied by subsequent work. Condone the omission of "d*h*" and "d*t*" provided the intention is clear but the minus sign must be present on one side or the other. | | B1 |
| | $20\ln h + 10\ln(30-h)$ | M1: Integrates their partial fractions to obtain $\pm P\ln h \pm Q\ln(30-h)$<br>A1: Correct integration for their partial fractions of the form $\frac{A}{h}+\frac{B}{30-h}$ following through their "*A*" and "*B*". | M1A1ft |
| | $t=0, h=10 \Rightarrow c = 20\ln 10 + 10\ln 20$ | Substitutes $h$ = 10 and $t$ = 0 to find a value for $c$. NB $c$ = 76.0… | M1 |
| | $h=5 \Rightarrow t = 20\ln 10 + 10\ln 20 - 10\ln 25 - 20\ln 5$<br>Substitutes $h$ = 5 and uses their value of $c$ to find a value for $t$. | | ddM1 |
| | $t = 11.63$ (secs) | Awrt 11.63 only | A1cso |
| | | | **(6)** |
| | | | **(14 marks)** |
| | **(c) Way 2** | | |
| | $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h(30-h)}{30(20-h)} \Rightarrow \int \frac{30(20-h)}{h(30-h)}\,\mathrm{d}h = -1\int \mathrm{d}t$<br>A correct statement which may be implied by subsequent work. Condone the omission of "d*h*" and "d*t*" provided the intention is clear but the minus sign must be present on one side or the other. | | B1 |
| | $20\ln h + 10\ln(30-h)$ | M1: Integrates their partial fractions to obtain $\pm P\ln h \pm Q\ln(30-h)$<br>A1: Correct integration for their partial fractions of the form $\frac{A}{h}+\frac{B}{30-h}$ following through their "*A*" and "*B*". | M1A1ft |
| | $(t=)\left[20\ln h + 10\ln(30-h)\right]_5^{10}$<br>or<br>$(t=)\left[20\ln h + 10\ln(30-h)\right]_{10}^{5}$ | Attempts the limits 5 and 10 for $h$. Either statement as shown is sufficient. | M1 |
| | $(t=)\left[20\ln 10 + 10\ln 20\right] - \left[20\ln 5 + 10\ln 25\right]$ | Substitutes $h$ = 5 and $h$ = 10 to find a value for $t$. | ddM1 |
| | $t$ = 11.63 | Awrt 11.63 only | A1cso |
| | | | **(6)** |

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